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Alg2/Trig
Notes [1.2] The Unit Circle

Name Key

Period ______

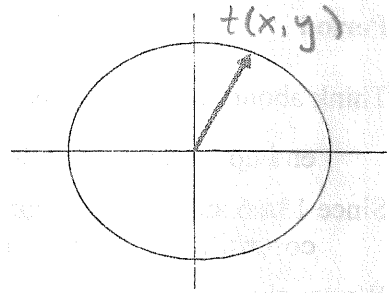
Recall the standard form of a circle: $(x-h)^2 + (y-k)^2 = r^2$

If the center is (0,0) and the radius = 1, we have the equation $x^2 + y^2 = 1$. This is called unit circle.

Recall the real number line:

Picture wrapping the real number line around the unit circle.

Each real number, t, corresponds to a point (x,y) on the circle. Also, each real number corresponds to a central angle.



$S = r\theta$ where $r = 1$ therefore $S = \theta$.

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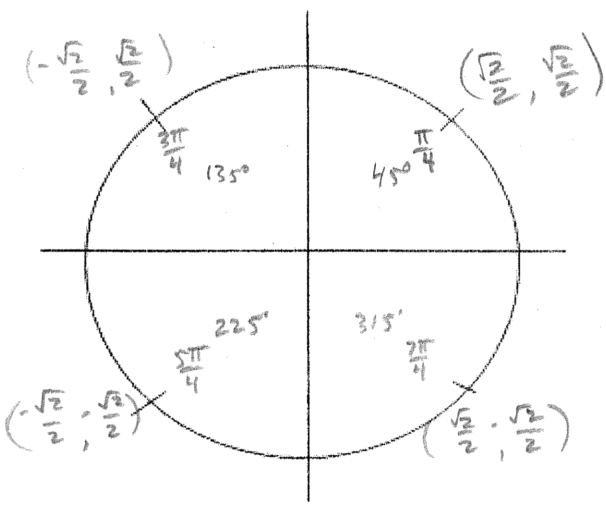
Definitions of Trig Functions

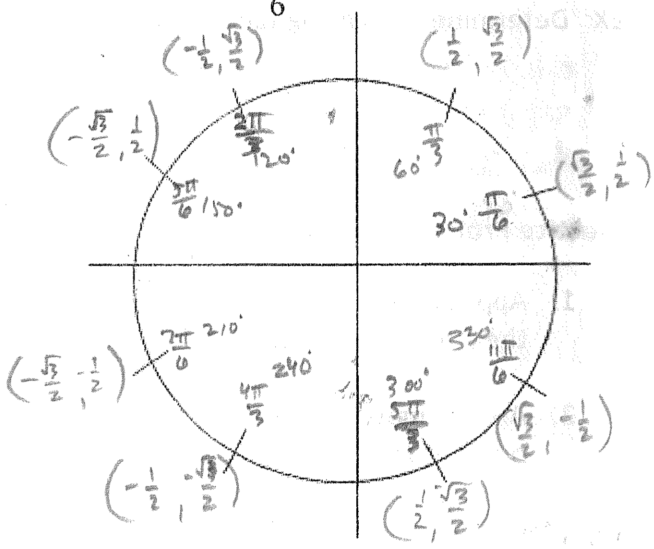
$\sin t = y$	$\csc t = \frac{1}{y} \quad (y \neq 0)$ $\quad \frac{1}{\sin t}$
$\cos t = x$	$\sec t = \frac{1}{x} \quad (x \neq 0)$ $\quad \frac{1}{\cos t}$
$\tan t = \frac{y}{x} \quad (x \neq 0)$	$\cot t = \frac{x}{y} \quad (y \neq 0)$ $\quad \frac{1}{\tan t}$

Know these common angles and their coordinates!

Multiples of $45°(\frac{\pi}{4})$



Multiples of $30°(\frac{\pi}{6})$



EX:

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Range

Notice that since $-1 \le y \le 1$ and sin t = y, we know $-1 \le \sin t \le 1$.

Also, since $-1 \le x \le 1$ and cos t = x, we know $-1 \le \cos t \le 1$.

These are the ranges of the sine and cosine functions.

Period

Think about the real number $t = \dfrac{13\pi}{6}$. As the real number line gets wrapped around the unit circle, $13\pi/6$ will end up on top of $\pi/6$. (Recall they are coterminal since $13\pi/6 - 2\pi = \pi/6$.)

Since $13\pi/6$ and $\pi/6$ both represent the same point on the circle, $\sin(13\pi/6) = \sin(\pi/6)$. Also, $\cos(13\pi/6) = \cos(\pi/6)$, $\tan(13\pi/6) = \tan(\pi/6)$, etc…

We say the period of sine and of cosine is 2π for this reason.

EX: Evaluate $\tan 5\pi$

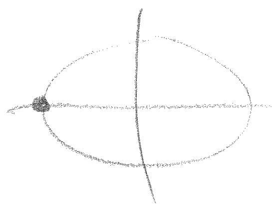
$5\pi - 2\pi - 2\pi$

$\tan \pi$

$= \dfrac{y}{x}$

$= \dfrac{0}{-1}$

$= 0$



EX: Evaluate $\cos -\dfrac{17\pi}{6}$

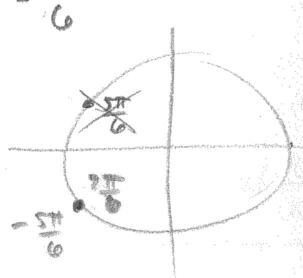
(2π)

$-\dfrac{17\pi}{6} + \dfrac{12\pi}{6}$

$\cos -\dfrac{5\pi}{6}$

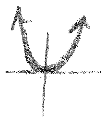
$= x$

$= -\dfrac{\sqrt{3}}{2}$



Even and Odd Trig Functions

First recall: Even $f(t) = f(-t)$



reflect

Odd $f(t) = -f(-t)$

rotate

EX: Determine if each trig function is even or odd or neither.

$\cos 30° = \cos(-30°)$ even

$\sin 30° = -\sin(-30°)$ odd

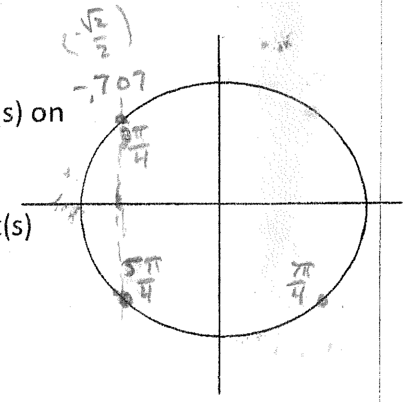
$\tan 30° = -\tan(-30°)$ odd

$\sec 30° = \sec(-30°)$ even

$\csc 30° = -\csc(-30°)$ odd

$\cot 30° = -\cot(-30°)$ odd

Practice Problems



1. Approximately where on the unit circle will the cosine be -.7? Place a point(s) on the circle to indicate this location. Label the point. $\dfrac{3\pi}{4}, \dfrac{5\pi}{4}$

2. Approximately where on the unit circle will the tangent be –1? Place a point(s) on the circle to indicate this location. Label the point. $\dfrac{y}{x}$ $\dfrac{3\pi}{4}, \dfrac{7\pi}{4}$

3. Given cos t = -3/4, find cos(-t) and sec(-t).
   $-\dfrac{3}{4}$ $\quad -\dfrac{4}{3}$

4. Evaluate $\tan\dfrac{9\pi}{4}$. $\quad \dfrac{9\pi}{4} - \underset{(2\pi)}{\dfrac{8\pi}{4}}$ $\quad \tan\dfrac{\pi}{4} = \dfrac{y}{x} = \dfrac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$

5. Find the point (x,y) on the unit circle that corresponds to $t = \dfrac{5\pi}{4}$. Label the point.
   $\left(-\dfrac{\sqrt{2}}{2}, -\dfrac{\sqrt{2}}{2}\right)$

Name Key

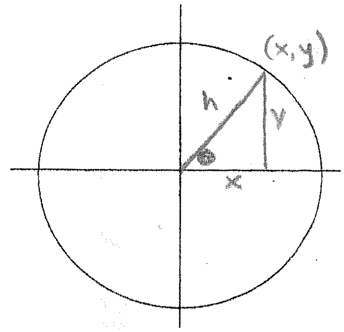
Period ______

Trigonometric functions can be defined as ratios of sides of a right triangle.

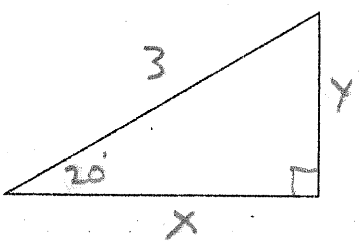
$$\cos\theta = \frac{x}{h} = \frac{adj}{hyp} \qquad \sec\theta = \frac{h}{x} = \frac{hyp}{adj}$$

$$\sin\theta = \frac{y}{h} = \frac{opp}{hyp} \qquad \csc\theta = \frac{h}{y} = \frac{hyp}{opp}$$

$$\tan\theta = \frac{y}{x} = \frac{opp}{adj} \qquad \cot\theta = \frac{x}{y} = \frac{adj}{opp}$$



EX: Find x and y.



$\sin 20° = \frac{y}{3}$

$\cdot 3 \quad = \cdot 3$

$y = 3\sin 20°$

$y = 1.026$

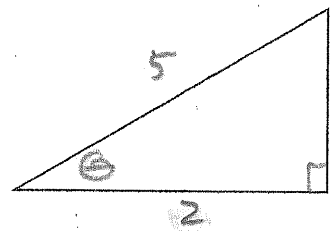
$\cos 20° = \frac{x}{3}$

$\cdot 3 \quad = \cdot 3$

$x = 3\cos 20°$

$x = 2.819$

EX: Find θ.



$\cos\theta = \frac{2}{5}$

$\cos^{-1} = \cos^{-1}$

$\theta = \cos^{-1}\frac{2}{5}$

$\theta = 66.4°$

Cofunctions

Cofunctions of complementary angles are equal.

$\sin(90° - \theta) = \cos\theta$
$\tan(90° - \theta) = \cot\theta$
$\sec(90° - \theta) = \csc\theta$

EX: 20° and 70° are complementary ∠s
sin & cos are cofunctions.

$\sin 20° \stackrel{?}{=} \cos 70°$

$.3420 = .3420$

EX: $\frac{\pi}{6}$ and $\frac{\pi}{3}$ are complementary ∠s
tan & cot are cofunctions

$\tan\frac{\pi}{6} \stackrel{?}{=} \cot\frac{\pi}{3}$

$\frac{y}{x}\ \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{x}{y}\ \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$

$\frac{\sqrt{3}}{3} = \frac{\sqrt{3}}{3}$

Fundamental Trig Identities

Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

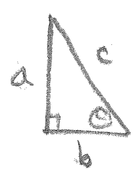
Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \cot^2\theta = \csc^2\theta \qquad \tan^2\theta + 1 = \sec^2\theta$$

now prove



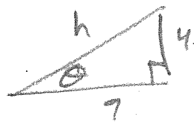
$\frac{a^2 + b^2}{c^2} = \frac{c^2}{c^2}$

$\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = 1$

$\boxed{\sin^2\theta + \cos^2\theta = 1}$

EX: Given $\tan\theta = \frac{4}{7}$

find $\sin\theta$ and $\cos\theta$



$$\sin\theta = \frac{4}{h} = \frac{4}{\sqrt{65}} = \frac{4\sqrt{65}}{65} \qquad \cos\theta = \frac{7}{h} = \frac{7}{\sqrt{65}} = \frac{7\sqrt{65}}{65}$$

$7^2 + 4^7 = h^2$

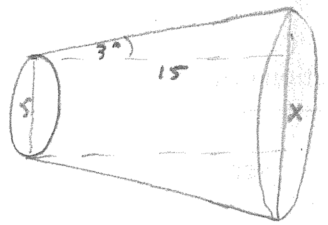
$16 + 49 = h^2$

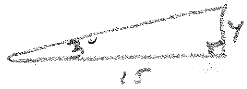
$65 = h^2$

$\sqrt{\ } = \sqrt{\ }$

$h = \sqrt{65}$

EX: Find x the diameter of large end.





$\tan 3° = \frac{y}{15}$

$15 \cdot \tan 3° = y$

$.7861 = y$

$x = 5 + 2(y)$

$x = 5 + 2(.7861)$

$x = 6.5722$

Practice Problems

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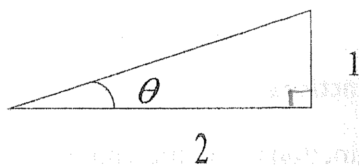
1) Find the exact values of the 6 trig functions of the angle θ.



$\sin\theta = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5} \qquad \csc\theta = \frac{\sqrt{5}}{1}$

$\cos\theta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5} \qquad \sec\theta = \frac{\sqrt{5}}{2}$

$\tan\theta = \frac{1}{2} \qquad \cot\theta = \frac{2}{1}$

$h^2 = 1^2 + 2^2$

$\sqrt{\ } = \sqrt{\ }$

$h = \sqrt{5}$

2) Given sec θ = 5, $\tan\theta = 2\sqrt{6}$. Use trig identities to find…

a) cos θ $= \frac{1}{5}$

b) cot θ $= \frac{\sqrt{6}}{12}$

c) cot (90 - θ) $= \tan\theta$ $= 2\sqrt{6}$

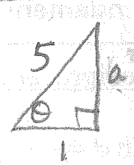
d) sin θ $= \frac{2\sqrt{6}}{5}$

$\sec\theta = 5$

$\tan\theta = 2\sqrt{6}$

$\boxed{\cos\theta = \frac{1}{5}}$

$\boxed{\cot\theta = \frac{1}{2\sqrt{6}} = \frac{\sqrt{6}}{12}}$



$a^2 + 1^2 = 5^2$

$-1 \quad -1$

$a^2 = 24$

$\sqrt{\ } = \sqrt{\ }$

$a = \sqrt{24}$

$= 2\sqrt{6}$