A40 / 1-41 eoo 45-67 odd

1. $\sin t \csc t = 1$

$= \sin t \cdot \frac{1}{\sin t}$

$= 1$

5. $\cos^2\beta - \sin^2\beta = 1 - 2\sin^2\beta$

$= 1 - \sin^2\beta - \sin^2\beta$

$= 1 - 2\sin^2\beta$

*9. $\frac{\sin^2\alpha - \sin^4\alpha}{\sin^2\alpha} = \cos^2\alpha - \cos^4\alpha$

$= \sin^2\alpha(1 - \sin^2\alpha)$

$= (1 - \cos^2\alpha)(\cos^2\alpha)$

$= \cos^2\alpha - \cos^4\alpha$

13. $\frac{\cot^2 t}{\csc t} = \csc t - \sin t$

$= \frac{\csc^2 t - 1}{\csc t}$

$= \csc t - \sin t$

17. $\frac{1}{\sec x \tan x} = \csc x - \sin x$

$= \frac{1}{\frac{1}{\cos x}\frac{\sin x}{\cos x}}$

$= \frac{\cos^2 x}{\sin x}$

$= \frac{1 - \sin^2 x}{\sin x}$

$= \csc x - \sin x$

21. $\cos x + \sin x \tan x = \sec x$

$= \cos x + \sin x \frac{\sin x}{\cos x}$

common den.

$= \frac{\cos^2 x}{\cos x} + \frac{\sin^2 x}{\cos x}$

$= \frac{1}{\cos x}(\cos^2 x + \sin^2 x)$

$= \sec x$

*25. $\frac{\cos\theta \cot\theta}{1 - \sin\theta} - 1 = \csc\theta$

$= \frac{\cos\theta \cot\theta (1 + \sin\theta)}{1 - \sin\theta(1 + \sin\theta)} - 1$

$= \frac{\frac{\cos^2\theta}{\sin\theta} + \cos^2\theta}{1 - \sin^2\theta} - 1$

$= \frac{\cos^2\theta(\frac{1}{\sin\theta} + 1)}{\cos^2\theta} - 1$

$= \csc\theta$

29. $\cos\left(\frac{\pi}{2} - x\right)\csc x = 1$

$= \sin x \cdot \frac{1}{\sin x}$

$= 1$

33. $\frac{\cos(-\theta)}{1 + \sin(-\theta)} = \sec\theta + \tan\theta$

$= \frac{\cos\theta}{1 - \sin\theta} \cdot \frac{1 + \sin\theta}{1 + \sin\theta}$

$= \frac{\cos\theta + \cos\theta\sin\theta}{\cos^2\theta}$

$= \sec\theta + \tan\theta$

37. $\frac{\tan x + \cot y}{\tan x \cot y} = \tan y + \cot x$

$= \frac{\tan x}{\tan x \cot y} + \frac{\cot y}{\tan x \cot y}$

$= \tan y + \cot x$

41. $\sin^2 x + \sin^2\left(\frac{\pi}{2} - x\right) = 1$

$= \sin^2 x + \cos^2 x$

$= 1$

45. $\dfrac{2\sec^2 x - 2\sec^2 x \sin^2 x - \sin^2 x - \cos^2 x}{-1} = 1$

$2\sec^2 x$

$= 2\sec^2 x(1 - \sin^2 x) - 1(\sin^2 x + \cos^2 x)$

$= 2\sec^2 x \cos^2 x - 1$

$= 2 - 1$

$= 1$

47. $2 + \cos^2 x - 3\cos^4 x = \sin^2 x(2 + 3\cos^2 x)$

$= (1 - \cos^2 x)(2 + 3\cos^2 x)$

$= 2 + 3\cos^2 x - 2\cos^2 x - 3\cos^4 x$

$= 2 + \cos^2 x - 3\cos^4 x$

49. $\csc^4 x - 2\csc^2 x + 1 = \cot^4 x$

$-1 \quad 1 \quad -1$

-2

$= (\csc^2 x - 1)(\csc^2 x - 1)$

$= \cot^2 x \quad \cot^2 x$

$= \cot^4 x$

51. $\sec^4\theta - \tan^4\theta = 1 + 2\tan^2\theta$

$= (\sec^2\theta + \tan^2\theta)(\sec^2\theta - \tan^2\theta)$

$= (1 + \tan^2\theta + \tan^2\theta)(1)$

$= 1 + 2\tan^2\theta$

53. $\dfrac{\sin\beta}{1 - \cos\beta} = \dfrac{1 + \cos\beta}{\sin\beta}$

$= \dfrac{\sin\beta}{1 - \cos\beta} \cdot \dfrac{1 + \cos\beta}{1 + \cos\beta}$

$= \dfrac{\sin\beta(1 + \cos\beta)}{\sin^2\beta}$

$= \dfrac{1 + \cos\beta}{\sin\beta}$

55.* $\dfrac{\tan^3\alpha - 1}{\tan\alpha - 1} = \tan^2\alpha + \tan\alpha + 1$

$a^3 - b^3$

$(a - b)(a^2 + ab + b^2)$

$= \dfrac{(\tan\alpha - 1)(\tan^2\alpha + \tan\alpha + 1)}{(\tan\alpha - 1)}$

$= \tan^2\alpha + \tan\alpha + 1$

57. $\ln|\tan\theta| = \ln|\sin\theta| - \ln|\cos\theta|$

$= \ln\dfrac{|\sin\theta|}{|\cos\theta|}$

$= \ln|\tan\theta|$

59.* $-\ln(1 + \cos\theta) = \ln(1 - \cos\theta) - 2\ln|\sin\theta|$

$= \ln(1 - \cos\theta) - \ln|\sin^2\theta|$

$= \ln\left(\dfrac{1 - \cos\theta}{1 - \cos^2\theta}\right)$

$= \ln\left(\dfrac{1 - \cos\theta}{(1 + \cos\theta)(1 - \cos\theta)}\right)$

$= \ln(1 + \cos\theta)^{-1}$

$= -\ln(1 + \cos\theta)$