Algebra 2/Trig
Ch2 Quadratic Functions – Review

Name Key

Period ____

Part1: Forms for Quadratics

:call the various forms of parabolas that we studied this chapter:

Vertex Form:	$y=a(x-h)^2+k$	*and*	$x=a(y-k)^2+h$
Alternate Vertex Form:	$y=\frac{1}{4p}(x-h)^2+k$	*and*	$x=\frac{1}{4p}(y-k)^2+h$
Intercept Form:	$y=a(x-p)(x-q)$		
Standard Form:	$y=ax^2+bx+c$		

No matter what form a parabola is written in, you should be able to identify the following key characteristics:

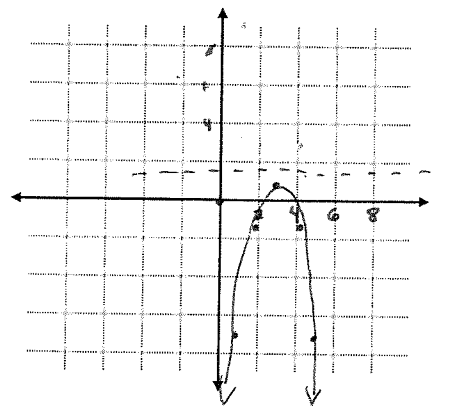
a) Vertex
b) Axis of symmetry
c) y-intercept
d) increasing interval
e) decreasing interval
f) The max or min value
g) Domain
h) Range

Problems 1-4: For each parabola, draw the graph then state each characteristic listed.

1. $y=-2(x-3)^2+1$
   a. Vertex $(3,1)$
   b. Axis of symmetry $x=3$
   c. y-intercept $(0,-17)$
   d. increasing interval $(-\infty,3)$
   e. decreasing interval $(3,\infty)$
   f. The max or min value max $=1$
   g. Domain $\mathbb{R}$
   h. Range $(-\infty,1]$
   i. P-value $-\frac{1}{8}$
   j. Focus $(3,\frac{7}{8})$
   k. Directrix $y=1\frac{1}{8}$

$\frac{1}{4p}=-2$

$-\frac{1}{8}=p$



2. $x=\frac{1}{8}(y-2)^2-4$
   a. Vertex $(-4,2)$
   b. Axis of symmetry $y=2$
   c. ~~y~~x-intercept $(-3\frac{1}{2},0)$
   ~~d. increasing interval~~
   ~~e. decreasing interval~~
   ~~f. The max or min value~~
   d ~~g~~. Domain $[-4,\infty)$
   e ~~h~~. Range $\mathbb{R}$
   f ~~i~~. P-value $p=2$
   g ~~j~~. Focus $(-2,2)$
   h ~~k~~. Directrix $x=-6$

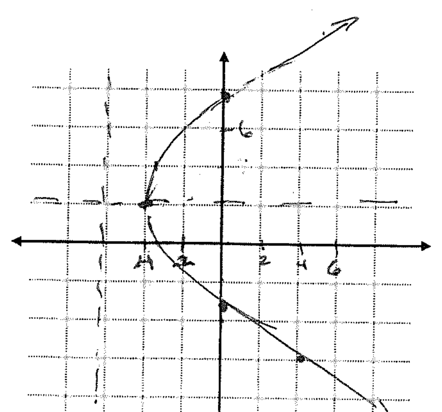
$x=\frac{1}{8}(0-2)^2-4$

$x=-3\frac{1}{2}$

$\frac{1}{4p}=\frac{1}{8}$

$8=4p$

$2=p$



Plot y-intercepts just to make sure its correct!

$0=\frac{1}{8}(y-2)^2-4$

$32=(y-2)^2$

$\pm\sqrt{32}=y-2$

$2\pm\sqrt{32}=y$

3. $y=\frac{1}{2}(x-1)(x+5)$

a. Vertex $(-2, -\frac{9}{2})$

b. Axis of symmetry $x=-2$

c. y-intercept $(0, -\frac{5}{2})$

d. increasing interval Incr: (-2, +oo)

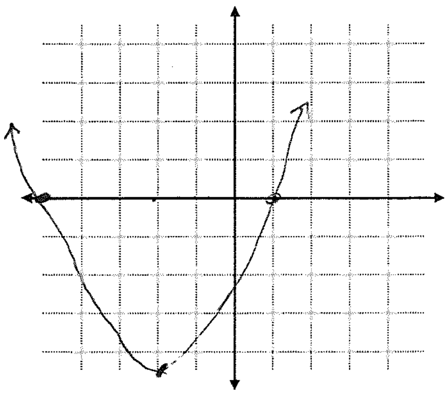
e. decreasing interval Decr: (-oo, -2)

f. The max or min value $y=-\frac{9}{2}$ min

g. Domain $\mathbb{R}$

h. Range $[-\frac{9}{2}, \infty)$

i. X-intercepts $(1,0)$ $(-5,0)$



4. $y=\frac{1}{2}x^2-x-4$

a. Vertex $(1, -\frac{9}{2})$

b. Axis of symmetry $x=1$

c. y-intercept $(0,-4)$

d. increasing interval Incr: (1, +oo)

e. decreasing interval Decr: (-oo, 1)

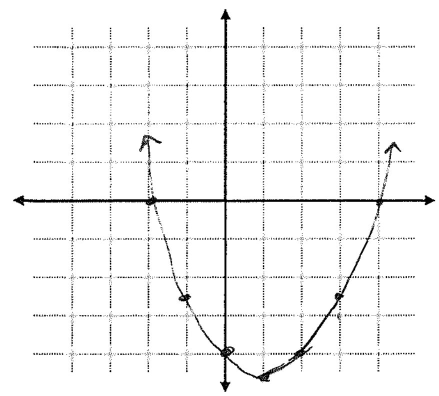
f. The max or min value min: $y=-\frac{9}{2}$

g. Domain $\mathbb{R}$

h. Range $[-\frac{9}{2}, \infty)$

$x=\frac{-b}{2a} = \frac{1}{2(\frac{1}{2})} = 1$

$y=\frac{1}{2}-1-4 = \frac{1}{2}-5 = \frac{-9}{2}$



Part 2: Transformations

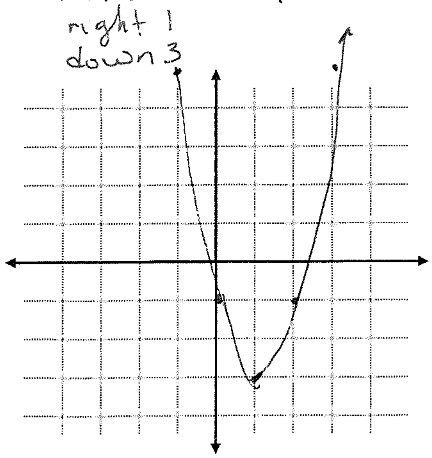
When there is more than one transformation on a function, sometimes the order matters. Specifically, if there is a vertical stretch or shrink or flip over x-axis paired with a vertical translation, or a horizontal stretch or shrink or flip over y-axis paired with a horizontal translation, the order will matter.

If given an equation in vertex form, always apply the vertical stretch or shrink and flip over the x-axis before translating by (h,k). Remember, vertex form won't have a horizontal stretch or shrink or flip over the y-axis (unless the parabola is opening sideways).

Problems 5-7: List in the correct order, the transformations that are performed on the parent graph. Then graph each.

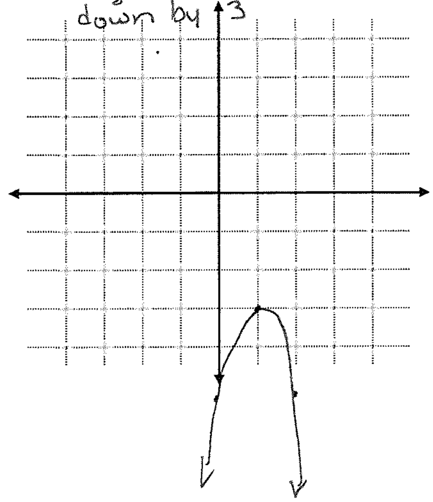
5. $y=2(x-1)^2-3$

vert stretch by 2
right 1
down 3



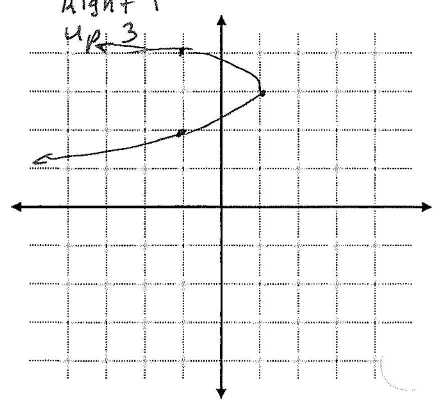
6. $y=-2(x-1)^2-3$

Vert. Stretch by 2
flip over x-axis
right by 1
down by 3



7. $x=-2(y-3)^2+1$

Horiz stretch by 2
Flip over y-axis
Right 1
up 3



When given a list of transformations to be performed, be sure to write out the resulting function after each transformation. This way, the order of the transformations will be taken into account.

Problems 8-10: Write a rule for $g(x)$ described by the transformations of the graph of $f(x)$. Then identify the vertex of $g(x)$ and graph $g(x)$.

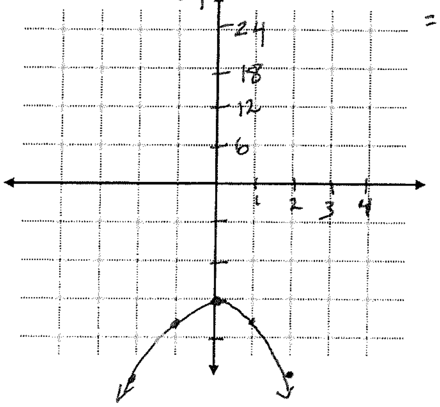
8. $f(x)=x^2+4$

Flip over x-axis. $g(x) = -f(x)$
$= -x^2-4$

Shift down by 2. $h(x) = g(x)-2$
$= -x^2-4-2$
$= -x^2-6$

Stretch vertically by 3. $i(x) = 3h(x)$
$= 3(-x^2-6)$
$= -3x^2-18$

vertex: $(0,4)$
$(0,-4)$
$(0,-6)$
$(0,-18)$



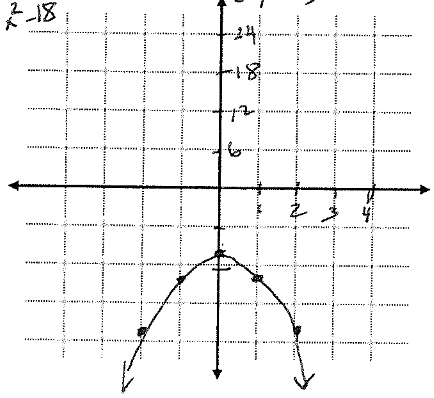
9. $f(x)=x^2+4$

Stretch vertically by 3. $g(x) = 3f(x)$
$= 3x^2+12$

Shift down by 2. $h(x) = g(x)-2$
$= 3x^2+10$

Flip over x-axis. $i(x) = -h(x)$
$= -3x^2-10$

Vertex: $(0,4)$
$(0,12)$
$(0,10)$
$(0,-10)$



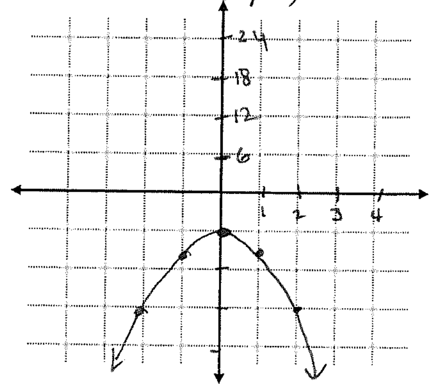
10. $f(x)=x^2+4$

Shift down by 2. $g(x) = f(x)-2$
$= x^2+2$

Flip over x-axis. $h(x) = -g(x)$
$= -x^2-2$

Stretch vertically by 3. $i(x) = 3h(x)$
$= -3x^2-6$

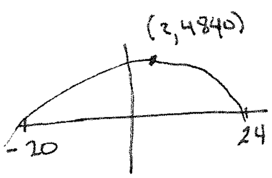
vertex: $(0,4)$
$(0,2)$
$(0,-2)$
$(0,-6)$



Part 3: Modeling with Quadratics

Problems 11-13: Word problems

11. A music store sells about 40 of a new model of drum per month at a price of $120 each. For each $5 decrease in price, about 2 more drums per month are sold. Write an equation that models the revenue from sales. State what the coordinates of the vertex are and what each coordinate of the vertex represents in the context of the problem. State the domain and range. (Rev) = (Price)(#sold)



$R(x) = (120-5x)(40+2x)$
$= -5(x-24)2(x+20)$
$= -10(x-24)(x+20)$

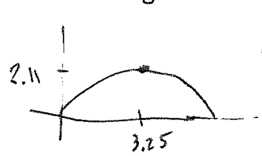
Axis of sym:
$x = \frac{-20+24}{2}$
$= 2$
$y = 4840$

Domain $0 \le x \le 24$
Range $0 \le y \le 4840$

max Revenue = $4840

vertex $(2, 4840) \to$ To max the revenue, drop price 2 times.

12. A woodland jumping mouse hops along a parabolic path given by $y=-0.2x^2+1.3x$ where x is the mouse's horizontal distance traveled (in feet) and y is the corresponding height (in feet). Can the mouse jump over a fence that is 3 feet height? Justify your answer.

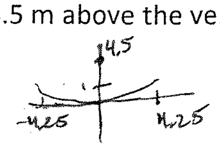


Axis of Sym: $x = \frac{-b}{2a}$
$= \frac{-1.3}{-.4}$
$= 3.25$

Vertex: $y = -.2(3.25)^2 + 1.3(3.25)$
$= 2.1125$
$(3.25, 2.11)$

max height is 2.11 feet so mouse can not jump over a 3 ft fence.

13. The Euro Dish, developed to provide electricity in remote areas, uses a parabolic reflector to concentrate sunlight onto a high-efficiency engine located at the reflector's focus. The sunlight heats helium to 650 degrees Celsius to power the engine. Write an equation for the Euro Dish's cross section with its vertex at (0,0). How deep is the dish if the focus is 4.5 m above the vertex and the width is 8.5m?



$p = 4.5$

$y = \frac{1}{4p}(x-h)^2 + k$

$y = \frac{1}{4(4.5)}(x-0)^2 + 0$

$y = \frac{1}{18}x^2$

$y = \frac{1}{18}(4.25)^2$

$= 1.00$ m depth

Part 4: Regression Equations

14. The following data represents the average maximum and minimum temperatures recorded each month in Raleigh, NC, over a 6-month period. The temperatures recorded are in degrees Fahrenheit.

Max x	72.3	79.0	85.2	88.2	87.1	81.6
Min y	46.5	55.3	62.6	67.1	68.0	60.4

(A) Find the QUADRATIC equation that best models the data.

$y = -.0069x^2 + 2.4480x - 94.523$

(B) Predict minimum temperature if the maximum temperature is 90.

$x = 90$

$y = -.0069(90)^2 + 2.4480(90) - 94.523$

$= 69.907$

(C) Predict the maximum temperature if the minimum temperature is 40.

$y = 40$

$40 = -.0069x^2 + 2.4480x - 94.523$ use intersect feature on calculator

$x = 67.977$

15. This data table shows the per capita consumption of broccoli, *b* (in pounds) for the years 1980 through 1989. Let *t* represent the year, with *t* = 0 corresponding to 1980.

Year x	1980 0	1981 1	1982 2	1983 3	1984 4	1985 5	1986 6	1987 7	1988 8	19
Pounds y	1.6	1.8	2.2	2.3	2.7	2.9	3.5	3.6	4.2	45

(A) Find the r^2 values for the linear and quadratic regressions of this data.

$r^2 = .98$ $r^2 = .99$

(B) Determine if the data is linear or quadratic. Why?

quad. regression (Has r^2 closer to 1.)

(C) Write the equation that best models the data.

$y = .011x^2 + .223x + 1.602$

(D) In which year was the per capita consumption of broccoli 5 pounds?

$y = 5$ $5 = .011x^2 + .223x + 1.602$ Find intersection on graph $(10.153, 5)$

10th year is 1990

(E) What would the per capita consumption of broccoli be in 2005?

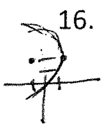
$x = 25$ $y = .011(25)^2 + .223(25) + 1.602$

$y = 14.052$ pounds

Part 5: Writing an Equation Given Key Points

Problems 16-20: Write an equation of a parabola that meets the given criteria. Use which ever form is most convenient.

16. Vertex: (1,2) Focus: (-1,2)



$x = \frac{1}{4p}(y-k)^2 + h$

$x = \frac{1}{4(-2)}(y-2)^2 + 1$

$x = \frac{1}{-8}(y-2)^2 + 1$

17. Vertex: (4,-5) Point: (2,-1) Opens Up

$y = a(x-h)^2 + k$

$y = a(x-4)^2 - 5$

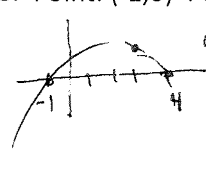
$-1 = a(2-4)^2 - 5$

$4 = a \cdot 4$

$1 = a$

$y = (x-4)^2 - 5$

18. Point: (-1,0) Point: (4,0) Point: (3,2)



$y = a(x+1)(x-4)$

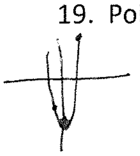
$2 = a(3+1)(3-4)$

$2 = a(4)(-1)$

$-\frac{1}{2} = a$

$y = -\frac{1}{2}(x+1)(x-4)$

19. Point: (-1,-3) Point: (0,-4) Point: (2,6) Opens Up



$y = ax^2 + bx - 4$

$-3 = a(-1)^2 + b(-1) - 4$

(1) $\boxed{1 = a - b}$

$6 = a(2)^2 + b(2) - 4$

$10 = 4a + 2b$

(2) $\boxed{5 = 2a + b}$

(1)+(2)

$6 = 3a$

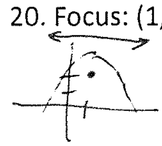
$2 = a$

$1 = 2 - b$

$b = 1$

$\boxed{y = 2x^2 + x - 4}$

20. Focus: (1,2) Directrix: y=8



$y = \frac{1}{4p}(x-h)^2 + k$

$y = \frac{1}{4(-3)}(x-1)^2 + 5$

$y = -\frac{1}{12}(x-1)^2 + 5$